

B.Sc. (Hons.) – III
Assignment – I
(Riemann Integration)

Time: 1 hour

Max. Marks: 25

(Attempt any five questions. All questions carry equal marks.)

1. Show that integral of sum of two bounded Riemann integrable functions on $[a, b]$ is the sum of their integrals. (5)
2. Give an example of an integrable function which has an infinite set of points of discontinuity having only one limit point. (5)
3. If f is Riemann integrable on $[a, b]$ show that the indefinite integral F of f given by
$$F(x) = \int_a^x f(t) dt, \quad \text{for all } x \in [a, b]$$
is uniformly continuous. (5)
4. Give example of a Riemann integrable function which is not monotonic. (5)
5. If f and g are two bounded and integrable functions on $[a, b]$ and there exists a number $k > 0$ such that $|g(x)| \geq k$ for all x in $[a, b]$, then prove that f/g is bounded and integrable in $[a, b]$. (5)
6. If f is bounded and integrable in the interval $[a, b]$ show that

$$\lim_{n \rightarrow \infty} \int_a^b f(x) \sin nx \, dx = 0 \quad (5)$$